Since the third and last factors are repeated, therefore deleting them once i.e., retaining one of them, we get the Product of Sums form as:

$$= (\bar{A} + B + C)(\bar{A} + \bar{B} + C)(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + \bar{C})(A + B + \bar{C})$$
$$= (100)(110)(101)(111)(001)$$
$$= \Pi(4,6,5,7,1)$$

$= \Pi(1,4,5,6,7)$ is the required POS form. To find the SOP form of $f(A,B,C)$ we multiply each factor by the absent variables $(C + \bar{C})$ and $(B + \bar{B})$ respectively.

Thus $f(A,B,C) = \bar{A}(B + \bar{C})$

$$= \bar{A}B + \bar{A}\bar{C}$$
$$= \bar{A}B(C + \bar{C}) + \bar{A}\bar{C}(B + \bar{B})$$
$$= \bar{A}BC + \bar{A}B\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C} \text{ (delete } \bar{A}B\bar{C} \text{ once as it is repeated twice)}$$
$$= \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C}$$
$$= 011 + 010 + 000$$
$$= \Sigma(3,2,0)$$

$= \Sigma(0,2,3)$ is the required SOP form.

EXAMPLE 33: Determine the canonical product of sums (POS) form and canonical sum of products (SOP) form of the switching functions: $f(A,B,C) = \bar{B} \cdot C$

Solution:

$$f(A,B,C) = \bar{B} \cdot C$$

The function has three-variables A, B, C. In the first factor $(\bar{B})$, the variables A and C are absent and in the second factor (C), the variables A and B are absent. Therefore, adding $A\bar{A} + C\bar{C}$ to the first factor and adding $A\bar{A} + B\bar{B}$ to the second factor, we get:

$$f(A,B,C) = (\bar{B} + A\bar{A} + C\bar{C}) \cdot (C + A\bar{A} + B\bar{B})$$
$$= [(\bar{B} + A) \cdot (\bar{B} + \bar{A}) + C\bar{C}][(C + A)(C + \bar{A}) + B\bar{B}]$$

[using distributive property]

$$= [(\bar{B} + A)(\bar{B} + \bar{A}) + C][(\bar{B} + A)(\bar{B} + \bar{A}) + \bar{C}][(C + \bar{A})(C + \bar{A}) + B]$$
$$[(C + A)(C + \bar{A}) + \bar{B}]$$

[again using distributive property]

$$= [C + (\bar{B} + A)(\bar{B} + \bar{A})][\bar{C} + (\bar{B} + A)(\bar{B} + \bar{A})][B + (C + A)(C + \bar{A})]$$
$$[\bar{B} + (C + A) + (C + \bar{A})]$$

Again by applying distributive property, we get

$$= (C + \bar{B} + A)(C + \bar{B} + \bar{A})(\bar{C} + \bar{B} + A)(\bar{C} + \bar{B} + \bar{A})(B + C + A)$$
$$(B + C + \bar{A})(\bar{B} + C + A)(\bar{B} + C + \bar{A})$$
$$= (A + \bar{B} + C)(\bar{A} + \bar{B} + C)(A + \bar{B} + \bar{C})(\bar{A} + \bar{B} + \bar{C})(A + B + C)$$
$$(\bar{A} + B + C)(A + \bar{B} + C)(\bar{A} + \bar{B} + C).$$

Retaining the repeated factors only once, i.e., by deleting the last two factors, we get.

$$= (A + \bar{B} + C)(\bar{A} + \bar{B} + C)(A + \bar{B} + \bar{C})(\bar{A} + \bar{B} + \bar{C})(A + B + C)(\bar{A} + B + C)$$

which is the required POS form, and further it can be written as $f(A,B,C)$

$$= (A + \bar{B} + C)(\bar{A} + \bar{B} + C)(A + \bar{B} + \bar{C})(\bar{A} + \bar{B} + \bar{C})(A + B + C)(\bar{A} + B + C)$$
$$= (010)(110)(011)(111)(000)(100)$$
$$= \Pi(2,6,3,7,0,4)$$
$$= \Pi(0,2,3,4,6,7) \text{ in POS form.}$$

To obtain the SOP form of the above function $f(A, B, C) = \bar{B}\,.\,C$, we multiply this factor by $A + \bar{A}$ as the variable A is missing

$$\therefore f(A,B,C) = \bar{B} \cdot C$$
$$= (A + \bar{A}) \cdot \bar{B}C$$
$$= A\bar{B}C + \bar{A}\bar{B}C$$
$$= 101 + 001$$
$$= \Sigma(5,1)$$
$$f(A,B,C) = \Sigma(1,5)$$

$\therefore A\bar{B}C + \bar{A}\bar{B}C$ or $\Sigma(1,5)$ is the required SOP form of the given function.

EXAMPLE 34: Find the minterms or canonical sum of products (SOP) form and maxterms or canonical product of sums (POS) form for function f(A,B,C)= AC+ AC.

Solution:

$$(A,B,C) = \bar{A}C + A\bar{C}$$

To obtain the minterms or SOP form of above function, we first examine all the terms of the expression and conclude that variable B is missing in both the terms. Therefore we multiply both these terms by $B + \bar{B}$

$$\therefore f(A,B,C) = \bar{A}C + A\bar{C}$$
$$= \bar{A}C(B + \bar{B}) + A\bar{C}(B + \bar{B})$$
$$= \bar{A}CB + \bar{A}C\bar{B} + A\bar{C}B + A\bar{C}\bar{B}$$
$$= \bar{A}BC + \bar{A}\bar{B}C + AB\bar{C} + A\bar{B}\bar{C}$$
$$= 011 + 001 + 110 + 100$$
$$= \Sigma(3,1,6,4)$$
$$= \Sigma(1,3,4,6)$$

$\therefore \bar{A}BC + \bar{A}\bar{B}C + AB\bar{C} + A\bar{B}\bar{C}$ are the required minterms and $\Sigma(1,3,4,6)$ is the canonical SOP form of the function. To obtain maxterms or POS form of the function $f(A,B,C) = \bar{A}C + A\bar{C}$, we first convert it into a product of sums.

$$f(A,B,C) = \bar{A}C + A\bar{C} + A\bar{A} + C\bar{C}$$
$$= \bar{A}C + A\bar{A} + A\bar{C} + C\bar{C}$$
$$= \bar{A}(C + A) + \bar{C}(A + C)$$
$$= (C + A)(\bar{A} + \bar{C})$$

Since variable B is missing in both of these, therefore we add $B\bar{B}$ to both these factors.

$$\therefore f(A,B,C) = (C + A + B\bar{B})(\bar{A} + \bar{C} + B\bar{B})$$
$$= (C + A + B)(C + A + \bar{B})(\bar{A} + \bar{C} + B)(\bar{A} + \bar{C} + \bar{B})$$
$$= (A + B + C)(A + \bar{B} + C)(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + \bar{C})$$
$$= (000)(010)(101)(111)$$
$$= \Pi(0,2,5,7)$$

$\therefore (A + B + C)(A + \bar{B} + C)(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + \bar{C}) = \Pi(0,2,5,7)$ are the required maxterms or canonical product of sums form of the given function.

EXAMPLE 35: Find the maxterms or canonical POS form of function $f(A,B,C)$, which is represented by the truth Table 2.13 given below:

Table 2.13

Decimal Value	*A*	*B*	*C*	*f*
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	1

Solution:

From the above table it is evident that the decimal values for which the function f assumes the value '0' are 0,3,5,6.

Thus by definition, function $f(A,B,C)$ is the product of these sum terms i.e.,

$$= \Pi(0,3,5,6) = (000)(011)(101)(110)$$

$\therefore f(A,B,C) = (A + B + C)(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)$ or $\Pi(0,3,5,6)$ are the required maxterms or canonical POS form of the given function

EXAMPLE 36: From the truth table given below in Table 2.14 express function f in sum of minterms and product of maxterms. Also obtain the simplified function $f(A,B,C)$ in the canonical sum of products (SOP) form and product of sums (POS) form.

Table 2.14

Decimal Value	*A*	*B*	*C*	*f*
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Solution:

From the above table it is clearly visible that the decimal values for which the function f assumes the value '1' are 1,2,5,6 = Σ (1,2,5,6)

$$\therefore f = 001 + 010 + 101 + 110$$
$$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}C + AB\bar{C}$$

The decimal values for which the function f assumes value '0' in the above table are 0,3,4,7.

$$= \Pi\,(0,3,4,7)$$
$$\therefore f = (000)\,(011)\,(100)\,(111)$$
$$= (A + B + C)\,(A + \bar{B} + \bar{C})\,(\bar{A} + B + C)\,(\bar{A} + \bar{B} + \bar{C})$$

Hence the sum of minterms = Σ (1,2,5,6)
Product of maxterms = Π (0,3,4,7)
Function f in SOP form = $\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}C + AB\bar{C}$
Function f in POS form = $(A + B + C)\,(A + \bar{B} + \bar{C})\,(\bar{A} + B + C)\,(\bar{A} + \bar{B} + \bar{C})$

EXAMPLE 37: Find the sum of minterms and function $f(A,B,C,D)$ in canonical sum of products form for the function $f(A,B,C,D) = A.C.$

Solution:

$f(A,B,C,D) = AC$

Since variables B and D are missing, therefore by multiplying it by $B + \bar{B}$ and $\bar{D} + D$, we get:

$$f(A,B,C,D) = AC(B + \bar{B})\,(D + \bar{D})$$
$$= (ACB + AC\bar{B})\,(D + \bar{D})$$
$$= ACBD + ACB\bar{D} + AC\bar{B}D + AC\bar{B}\bar{D}$$
$$= ABCD + ABC\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D}$$

which is the required sum of minterms of function.

$$f(A,B,C,D) = ABCD + ABC\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D}$$
$$= 1111 + 1110 + 1011 + 1010$$

[$\therefore$ in SOP form an uncomplemented variable is represented by '1' and a complemented variable by '0']

$$= \Sigma\,(15,14,11,10)$$
$$= \Sigma\,(10,11,14,15)$$

which is the required SOP form of the switching function.

EXAMPLE 38: Express $f(A,B,C,D) = AB + \bar{A}BC + \bar{C}D$ as the sum of minterms and as the product of maxterms.